Module 3c: Flow in Pipes
Hazen-Williams Equation

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Hazen-Williams Equation

- Based on experimental work
- Used to calculated velocity in a pipe based on the relative roughness and slope of the energy line

$$V = kCR^{0.63}S^{0.54}$$

Where V = velocity
C = factor for relative roughness
R = hydraulic radius
S = slope of the energy line (head loss divided by pipe length)
k = "conversion" factor for unit system
k = 0.849 for units of m/sec
k = 1.318 for units of ft/sec

Hazen-Williams Equation

Type of Pipe	C
Pipes extremely straight and smooth	140
Pipes very smooth	130
Smooth wood, smooth masonry	120
New riveted steel, vitrified clay	110
Old cast iron, ordinary brick	100
Old riveted steel	95
Old iron in bad condition	60 – 80

In: Metcalf & Eddy, Inc. and George Tchobanoglous. *Wastewater Engineering: Collection and Pumping of Wastewater*. McGraw-Hill, Inc. 1981. Table 2-2.

Hazen-Williams Equation

Pipe Material	C
New cast iron	130
5-year old cast iron	120
20-year old cast iron	100
Average concrete	130
New welded steel	120
Asbestos cement	140
Plastic	150

In: Warren Viessman, Jr. and Mark Hammer. *Water Supply and Pollution Control, Sixth Edition*. Addison-Wesley. 1998. Table 6-1.

Hazen-Williams Equation

Description of Pipe	Value of C
Cast iron	130
New	130
5 years old	120
10 years old	110
20 years old	90 – 100
30 years old	75 – 90
Concrete	120
Cement lined	140
Welded steel	As for cast iron 5 years older
Riveted steel	As for cast iron 10 years older
Plastic	150
Asbestos Cement	140

In: Terence McGhee. *Water Supply and Sewerage, Sixth Edition*. McGraw-Hill. 1991.

Channel Characteristics: Hydraulic Radius and Wetted Perimeter

- Wetted Perimeter – surface of pipe or channel where fluid is touching (accounts for areas where friction effects are occurring).
  - Not used to describe any area that is open to the atmosphere since friction contact with the atmosphere is negligible.

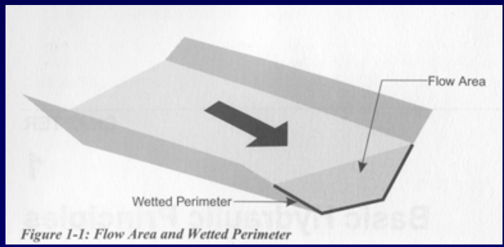


Figure 1-1: Flow Area and Wetted Perimeter

Channel Characteristics: Hydraulic Radius and Wetted Perimeter

- Hydraulic Radius, R:

$$R = \frac{A}{P}$$

Where A = cross-sectional area of flow
P = wetted perimeter

- Hydraulic radius will have units of length.

→For circular pipes flowing full, the hydraulic radius is:

A = area = $\pi D^2/4$

P = circumference = πD

R = A/P = $(\pi D^2/4)/(\pi D) = D/4$

Hazen-Williams Equation

- For circular pipes flowing full, the Hazen-Williams formula can be restated:

For a circular pipe flowing full:

$$R = \frac{D}{4}$$

By Continuity:

$$Q = VA$$

Hazen-Williams Equation

- For circular pipes flowing full, the Hazen-Williams formula can be restated:
  - Conversion Factor:

$$1\,ft3/\sec(7.48\,gal/\,ft3)=7.48\,gal/\sec$$
$$7.48\,gal/\sec(60\sec/\min)=448\,gal/\min$$
$$448\,gal/\min(60\min/hr)(24hr/day)=646{,}272\,gal/day$$
$$646{,}272\,gal/day\cong 0.646\text{ million gal/day}$$
$$\text{Conversion Factor}=0.646\,MGD/cfs$$

Hazen-Williams Equation

- Substituting into Hazen-Williams:

$$V[ft/\sec]=1.318CR0.63S0.54$$
$$V[ft/\sec]A[ft^2]=1.318ACR0.63S0.54$$
$$Q[ft^3/\sec]=1.318\frac{\pi}{4}D^2C\left(\frac{D}{4}\right)^{0.63}S^{0.54}$$
$$Q=1.035D^2\left(\frac{D}{4}\right)^{0.63}CS^{0.54}$$
$$Q=\left(\frac{1.035}{4^{0.63}}\right)CD^{2.63}S^{0.54}$$
$$Q=0.432CD^{2.63}S^{0.54}\;\;[\text{units of ft}^3\text{/sec and ft}]$$

Hazen-Williams Equation

- Substituting into Hazen-Williams:

$$Q=0.432CD^{2.63}S^{0.54}\;[\text{units of ft}^3\text{/sec}]$$
$$Q=0.432CD^{2.63}S^{0.54}(0.646MGD/cfs)$$
$$Q=0.279CD^{2.63}S^{0.54}$$

Where Q = flow (MGD)
D = diameter (ft)

Hazen-Williams Equation

- For SI units, the Hazen-Williams equation for pipes flowing full:

$$Q=0.278CD^{2.63}S^{0.54}$$

Where Q = flow (m^3/sec)
D = pipe diameter (m)

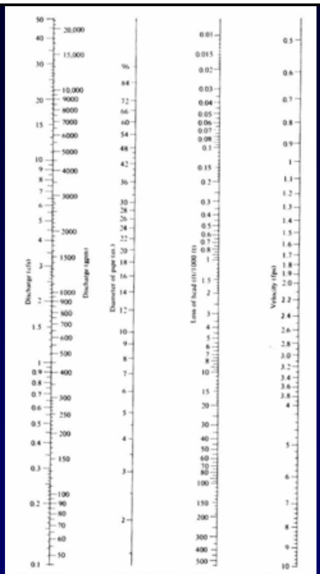


Figure 5.1 Nomograph for the Hazen–Williams formula with C = 100.

From: *Water Supply and Pollution Control, Sixth Edition*. Warren Viessman, Jr., and Mark J. Hammer. Addison Wesley, 1998.

Hazen-Williams Equation

Example:

- Determine the head loss in a 1000-m pipeline with a diameter of 500 mm that is discharging 0.25 m^3/sec. Assume that the Hazen-Williams coefficient for the pipe equals 130.

  Given: L = 1000 m
  D = 0.5 m
  Q = 0.25 m^3/sec
  C = 130

Hazen-Williams Equation

- Using the Hazen-Williams equation for flow:

$$Q = 0.278CD^{2.63}S^{0.54}$$

- By definition:

$$S = \frac{h_f}{L}$$

- Substituting:

$$Q = 0.278CD^{2.63}\left(\frac{h_f}{L}\right)^{0.54}$$

Hazen-Williams Equation

- Solving for h_f:

$$\frac{Q}{0.278CD^{2.63}} = \left(\frac{h_f}{L}\right)^{0.54}$$

$$\frac{h_f}{L} = \left(\frac{Q}{0.278CD^{2.63}}\right)^{1/0.54}$$

$$h_f = L\left(\frac{Q}{0.278CD^{2.63}}\right)^{1/0.54}$$

Substituting :

$$h_f = (1000m)\left(\frac{(0.25m^3/\sec)}{0.278(130)(0.5m)^{2.63}}\right)^{1.85}$$

$$h_f = 2.94m$$

Hazen-Williams Equation

Example:

- A 14-inch diameter schedule 80 pipe has an inside diameter of 12.5 inches (317.5 mm). What is the friction factor *f* if the pipe is flowing full and the allowable head loss is 3.5 m in a length of 200 m? Use Hazen-Williams equation to calculate velocity.

$$V = kCR^{0.63}S^{0.54}$$

Hazen-Williams Equation

- Assume that the pipe is cast-iron ⇒ C = 120
- Calculate the slope of the energy gradient:

$$S = \frac{h_f}{L} = \frac{3.5m}{200m}$$

$$S = 0.0175$$

- Calculate the hydraulic radius, R, for pipe flowing full:

$$R = \frac{D}{4} = \frac{0.3175m}{4} = 0.079375m$$

Hazen-Williams Equation

- Substituting into the Hazen-William equation:
  - For SI units, k = 0.849

$$V = (0.849)CR^{0.63}S^{0.54}$$

$$V = (0.849)(120)(0.079375m)^{0.63}(0.0175)^{0.54}$$

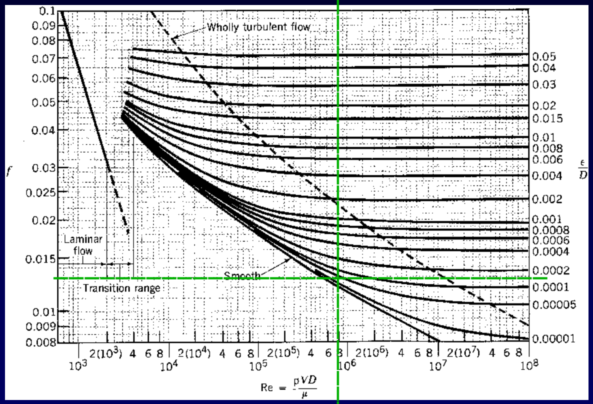
$$V = 2.32m/\sec$$

- Calculate a Reynolds number based on this velocity:

$$\mathrm{Re} = \frac{VD}{\nu} = \frac{2.32m/\sec(0.3175m)}{1.003x10^{-6}m^2/\sec}$$

$$\mathrm{Re} = 7.3x10^5$$

Refer to Moody Diagram



Reading from the Moody diagram (since this is based on a previous problem, *ε/d* has already been calculated and is 0.000012):

f = 0.0125

Hazen-Williams Equation

To correct for C factors not equal to 100 when using the Hazen-Williams nomogram:

- Given flow and diameter, find S_{100} from the nomogram:

$$S_C = S_{100}\left(\frac{100}{C}\right)^{1.85}$$

- Given flow and slope, find D_{100} from the nomogram:

$$D_C = D_{100}\left(\frac{100}{C}\right)^{0.38}$$

- Given diameter and slope, find flow from nomogram:

$$Q_C = Q_{100}\left(\frac{C}{100}\right)$$

Hazen-Williams Equation

Example (using the nomogram and the conversions):

- A schedule 80 pipe has an inside diameter of 12.5 inches (317.5 mm). What is the velocity if the pipe is flowing full and the allowable head loss is 3.5 m in a length of 200 m? Use Hazen-William equation to calculate velocity. Assume C = 120.

- Given: D = 12.5 inches
  H_L = 3.5 m
  L = 200 m

Hazen-Williams Equation

- Calculate slope of energy line.

$$S = \frac{H_L}{L} = \frac{3.5m}{200m}$$
$$S = 0.0175$$

- To use the nomogram, need slope as feet/1000 feet.

$$S = 0.0175\, ft/ft\left(\frac{1000\, ft}{1000\, ft}\right)$$
$$S = 17.5\, ft/1000\, ft$$

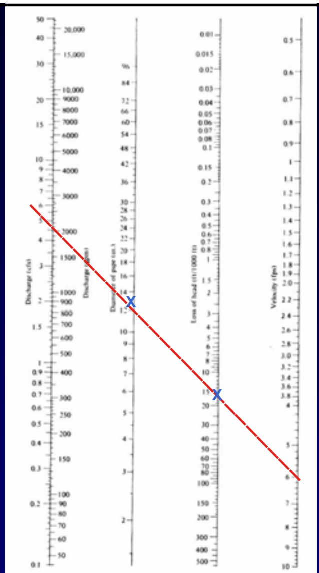


Figure 5.1 Nomograph for the Hazen–Williams formula with C = 100.

From the nomogram,

V = 6.0 ft/sec (0.3048 m/ft)

V = 1.83 m/sec

Hazen-Williams Equation

- Modifying based on flow conversion (divide both sides by area) and substituting:

$$Q_C = Q_{100}\left(\frac{C}{100}\right)$$

$$V = \frac{Q}{A}$$

$$V_C = \frac{Q_C}{A} = Q_{100}\left(\frac{C}{100}\right) = \frac{V_{100}}{A}\left(\frac{C}{100}\right)$$

$$V_C = V_{100}\left(\frac{C}{100}\right)$$

$$V_C = (1.83\,m/\sec)\left(\frac{120}{100}\right)$$

$$V_{120} = 2.2\,m/\sec$$

Hazen-Williams Equation

Example:

- Determine the head loss in a 46-cm concrete pipe with an average velocity of 1.0 m/sec and a length of 30 m.
- Using Hazen-Williams equation:

$$V = kCR^{0.63}S^{0.54}$$

- Since in SI units, k = 0.849
  - For a pipe flowing full:

$$R = \frac{D}{4} = \frac{0.46m}{4}$$

$$R = 0.115m$$

Hazen-Williams Equation

- By definition:
  Slope of energy line = Head Loss/Length of Pipe
  Or
  Head Loss = h_L = (Slope)(Pipe Length)
- Let C = 130 (concrete pipe ⇒ use average of 120 – 140)
- Solve Hazen-Williams for Slope:

$$V = kCR^{0.63}S^{0.54}$$

$$S^{0.54} = \frac{V}{kCR^{0.63}}$$

$$S = \left(\frac{V}{kCR^{0.63}}\right)^{1/0.54}$$

Hazen-Williams Equation

- Substituting:

$$S = \left(\frac{V}{kCR^{0.63}}\right)^{1/0.54}$$

$$S = \left(\frac{1m/\sec}{0.849*130*0.115^{0.63}}\right)^{1.85}$$

$$S = 0.0015$$

Head Loss = Slope(Pipe Length)

H_L = 0.0015(30 m) = 0.045 m

Hazen-Williams Equation

Example:

- Find the discharge from a full-flowing cast iron pipe 24 in. in diameter having a slope of 0.004.
- Assume: 5-year old cast iron ⇒ C = 120
- Using Hazen-Williams:

$$Q = 0.279CD^{2.63}S^{0.54}$$

- The flow will be given in MGD, but it is needed in ft^3/sec.
- From earlier, conversion factor for MGD to cfs is:

  1 ft^3/sec = 0.646 MGD

Hazen-Williams Equation

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Hazen-Williams Equation

- Modify Hazen-Williams:

$$Q = 0.279CD^{2.63}S^{0.54}\left(\frac{1\,ft^3/\sec}{0.646MGD}\right)MGD$$

$$Q = 0.432CD^{2.63}S^{0.54}$$

  for flow in ft^3/sec

- Substituting:

$$Q = 0.432(120)\left(\frac{24in}{12in/\,ft}\right)^{2.63}(0.004)^{0.54}$$

$$Q = 16.27cfs$$